

**Journal of Analysis and Applications**

Vol. 19 (2021), No.2, pp.119-134

ISSN: 0972-5954

© SAS International Publications

URL : www.sasip.net

# Nullity and deficiency of linear relations under certain perturbations\*

S.L. Kito and G. Wanjala<sup>†</sup>

---

**Abstract.** Let  $\mathcal{A}$  and  $\mathcal{B}$  be two closed linear relations acting between two Banach spaces  $X$  and  $Y$  and let  $\lambda$  be a complex number. We study the behavior of the nullity and deficiency of  $\mathcal{A}$  when perturbed by  $\lambda\mathcal{B}$ .

In particular, we show the existence of a constant  $\rho > 0$  for which both the nullity and deficiency of  $\mathcal{A}$  do not remain constant when  $\mathcal{A}$  is perturbed by  $\lambda\mathcal{B}$  for all  $\lambda$  inside the disk  $|\lambda| < \rho$ . It turns out, however, that these quantities do not depend on  $\lambda$  in the specified disk, that is, both the nullity and deficiency of  $\mathcal{A} - \lambda\mathcal{B}$  are uniform on the specified disk.

**AMS Subject Classification (2010):** Primary 47A06, 47A53, 47A55

**Keywords:** Linear relation, nullity, deficiency, non-stability

---

---

\*This work was financially supported by the Swedish Sida Phase-IV bilateral program with Makerere University, 2015-2020, project 316 Capacity building in Mathematics and its applications".

<sup>†</sup>Corresponding author